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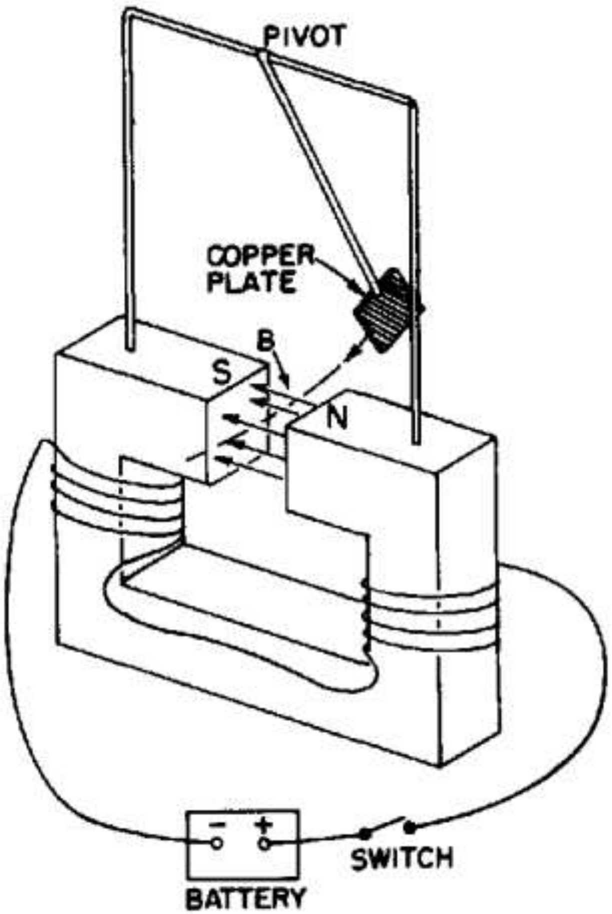
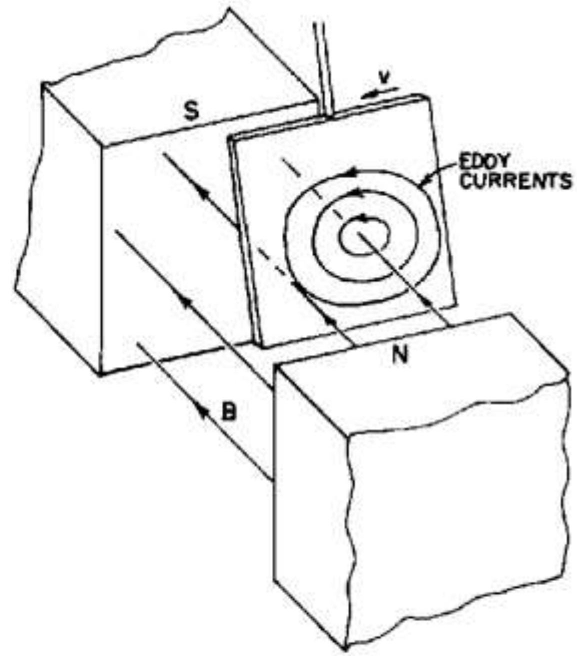
Time: 9 am

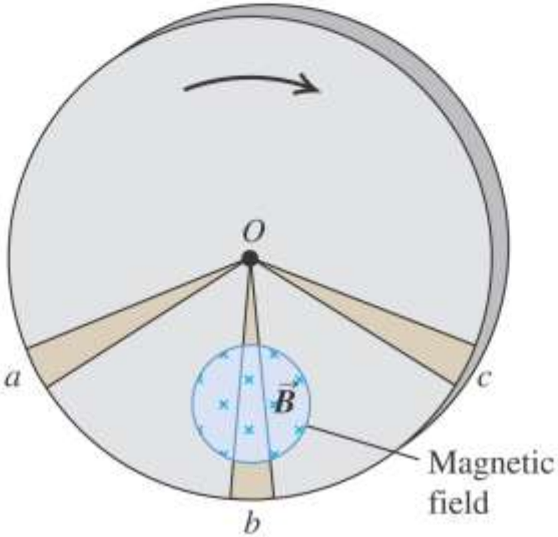
Eddy Currents



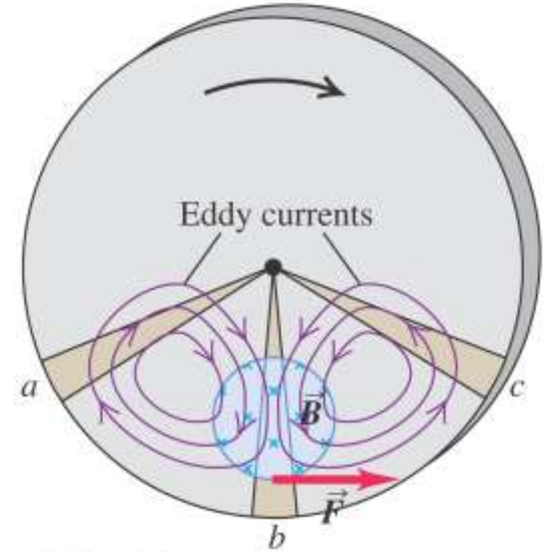


When magnetic field is on, currents (eddy currents) are induced in conductors so that the pendulum slows down or stops

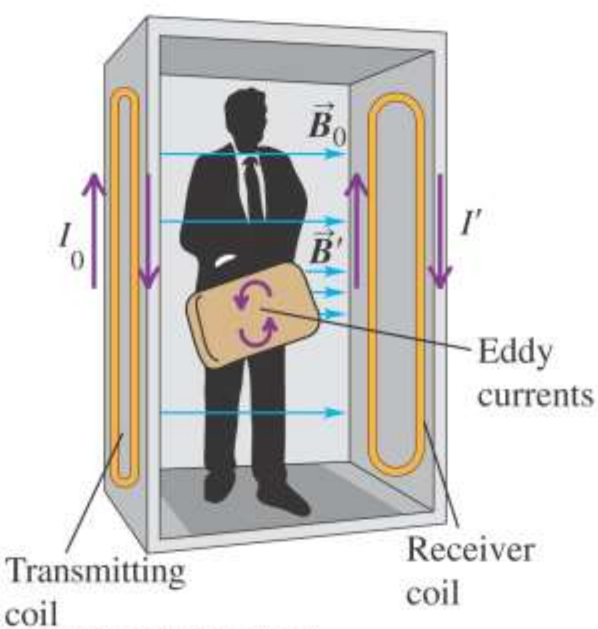
(a) Metal disk rotating through a magnetic field

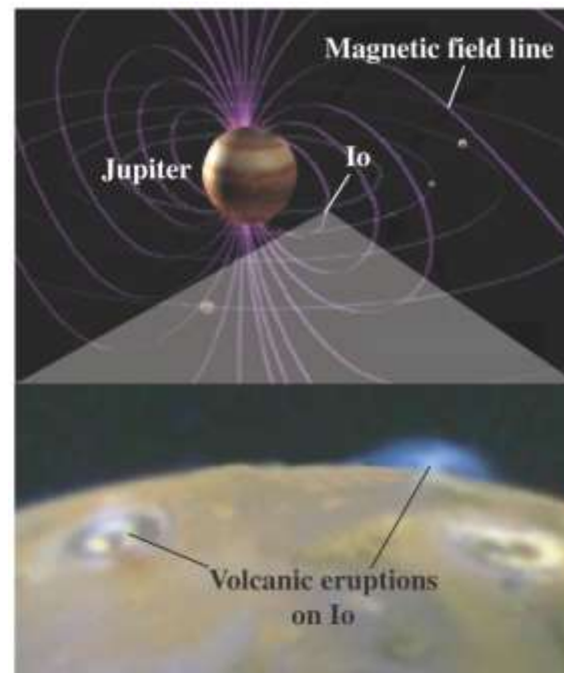


(b) Resulting eddy currents and braking force

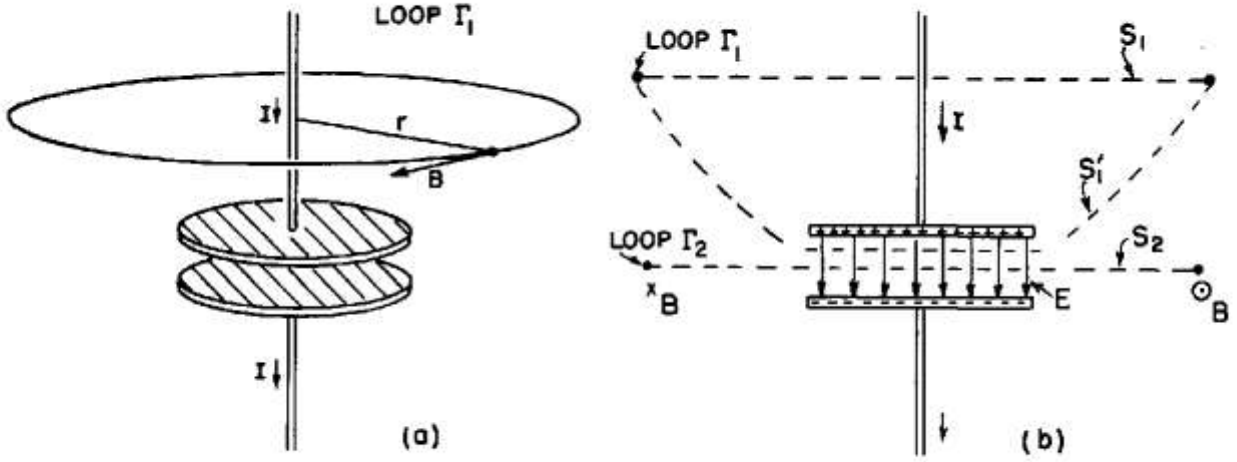


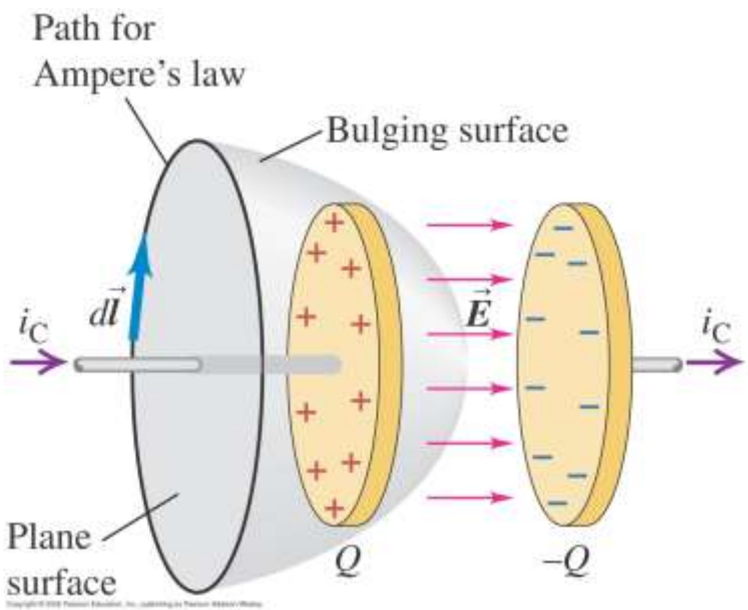
(a)





Displacement Current





$$\oint B \cdot dl = \mu_0 I_{encl}$$

$$q = Cv = \frac{\varepsilon_0 A}{d}(Ed) = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$i_c = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} - \text{displacement current}$$

Inadequacy of Ampere's Law for time-varying currents:

$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$ becomes contradictory

once applied to non-steady currents

Its generalization to one of the Maxwell equations

is a great example of a purely theoretical analysis

of the consistency of theory culminating in a result

with far-reaching consequences

Maxwell's generalization: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \dfrac{\partial \Phi_E}{\partial t}$

"displacement current" of the <u>electric field</u> flux as opposed to conduction current

Not only currents but changing electric fields too

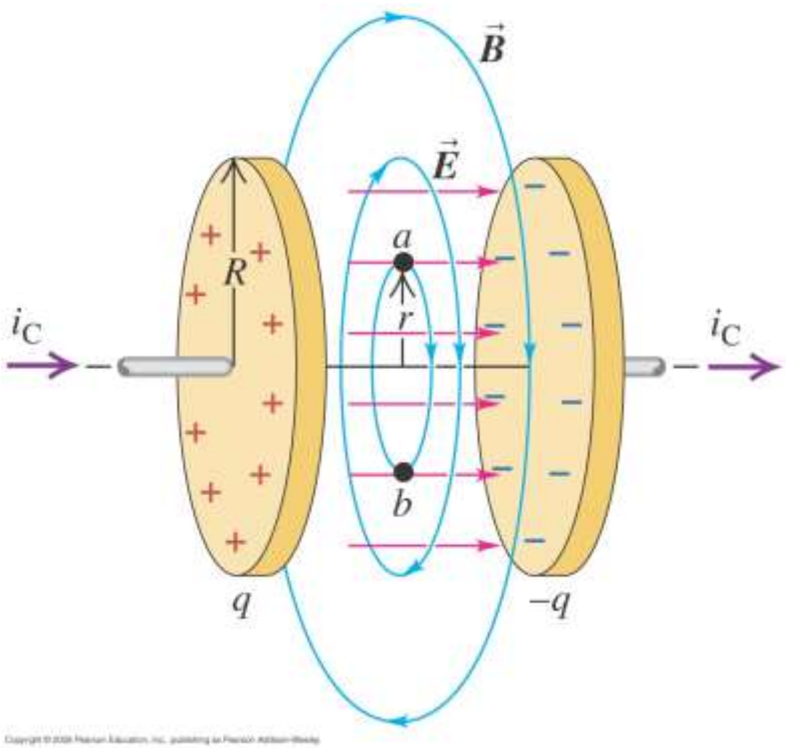
give rise to circulating magnetic fields!!

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

$$c^2 \nabla \times \mathbf{B} = \frac{\mathbf{j}}{\varepsilon_0} \quad \rightarrow \quad c^2 \nabla \times \mathbf{B} = \frac{\mathbf{j}}{\varepsilon_0} + \frac{\partial \mathbf{E}}{\partial t}$$

The Reality of Displacement Current

$$i_D = i_c$$



$$\oint B \cdot dl = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_D$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_c - \text{inside the capacitor}$$

$$B = \frac{\mu_0}{2\pi r} i_c - \text{ outside capacitor}$$

Field in the region outside of the capacitor exists
as if the wire were continuous *within* the capacitor

Maxwell equations in all their consistency and beauty

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$ Gauss's Law for ***E***

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_\Gamma \mathbf{E} \cdot dl = -\frac{\partial}{\partial t}\left(\int \mathbf{B} \cdot d\mathbf{A}\right)$$ Faraday's Law

$$\nabla \times \mathbf{B} = \frac{\mathbf{j}}{c^2 \varepsilon_0} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\oint_\Gamma \mathbf{B} \cdot dl = \mu_0\left(I_c + \varepsilon_0 \frac{\partial}{\partial t}\left(\int \mathbf{E} \cdot d\mathbf{A}\right)\right)$$ Ampere's Law

$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_S B \cdot d\mathbf{A} = 0$$ Gauss's Law for ***B***

differential form

integral form

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

How are these equivalent?

Use Divergence and Curl Theorems

Special cases of the more general Stokes' theorem

The Divergence theorem relates the flow (flux) of a vector field through a surface to the behavior of the vector field inside the surface.

More precisely: the outward flux of a vector field through a closed surface is equal to the volume integral of the divergence of the region inside the surface.

the sum of all sources minus the sum of all sinks gives the net flow out of a region.

$$\int_V (\nabla \cdot \mathbf{F})\, dV = \oint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

The left side is a volume integral over the volume V, the right side is the surface integral over the boundary of the volume V.

The Curl Theorem relates the surface integral of the curl of a vector field over a surface S to the line integral of the vector field over its boundary,

$$\oint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \int \mathbf{F} \cdot d\mathbf{s}$$

The left side is a surface integral and the right side is a line integral

Equivalence of integral and differential forms of Gauss's law for electric fields

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\varepsilon_0}$$

If ρ is the charge density (C/m^3), the total charge in a volume is the integral over that volume of ρ

$$Q = \int_V \rho dV$$

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \int_V \frac{\rho}{\varepsilon_0} dV$$

But from the divergence theorem:

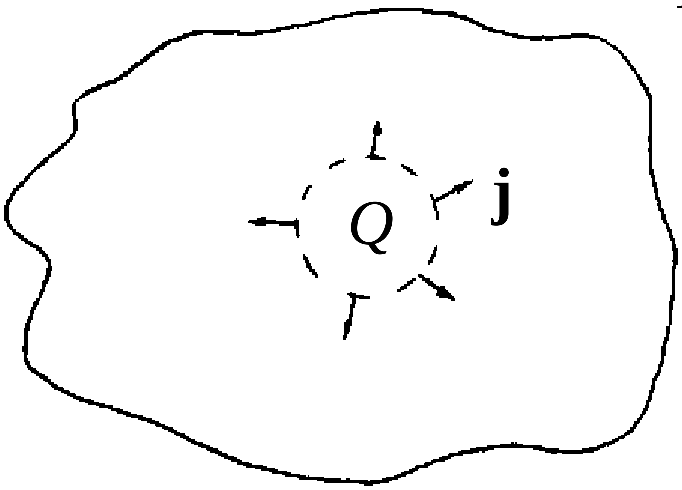
$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \oint_S \mathbf{E} \cdot \mathbf{n} dS = \int_V (\nabla \cdot \mathbf{E}) dV$$

$$\int_V (\nabla \cdot \mathbf{E}) dV = \int_V \frac{\rho}{\varepsilon_0} dV$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

It is often written as $\nabla \cdot \mathbf{D} = \rho$ where $\mathbf{D} = \varepsilon_0 \mathbf{E}$

Something very fundamental



Charge conservation

Because of the current through the closed surface, the charge inside changes with time :

$$-\frac{\partial Q}{\partial t} = I$$

$$-\frac{\partial}{\partial t}\left(\int \rho \, dV\right) = \oint \mathbf{j} \cdot \mathbf{n} \, dA = \int \nabla \cdot \mathbf{j} \, dV$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Just Ampere's law would be inconsistent with charge conservation:

$$\nabla\cdot(c^2\nabla\times\mathbf{B})\equiv 0=\frac{1}{\varepsilon_0}\nabla\cdot\mathbf{j} \quad \text{- Where's } \frac{\partial\rho}{\partial \mathrm{t}}\text{??}$$

Maxwell's modification restores consistency:

$$\nabla\cdot(c^2\nabla\times\mathbf{B})\equiv 0=\nabla\cdot\left(\frac{\mathbf{j}}{\varepsilon_0}+\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\varepsilon_0}$$

$$=\frac{1}{\varepsilon_0}\nabla\cdot\mathbf{j}+\frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{E}\right)=\frac{1}{\varepsilon_0}\left(\nabla\cdot\mathbf{j}+\frac{\partial\rho}{\partial t}\right)=0 \quad !!!$$

So the next (?) time you see a shirt that looks like this:

You will know what it means!

Maxwell equations and electromagnetic waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$ Gauss's Law for ***E***

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_\Gamma \mathbf{E} \cdot dl = -\frac{\partial}{\partial t}\left(\int \mathbf{B} \cdot d\mathbf{A}\right)$$ Faraday's Law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\oint_\Gamma \mathbf{B} \cdot dl = \mu_0 \left(I_c + \varepsilon_0 \frac{\partial}{\partial t}\left(\int \mathbf{E} \cdot d\mathbf{A}\right)\right)$$ Ampere's Law

$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_S B \cdot d\mathbf{A} = 0$$ Gauss's Law for ***B***

differential form | integral form

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

Electromagnetic disturbances in free space

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$c^2 \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

No charges and no currents!

With a complete set of Maxwell equations, a remarkable new phenomenon occurs:

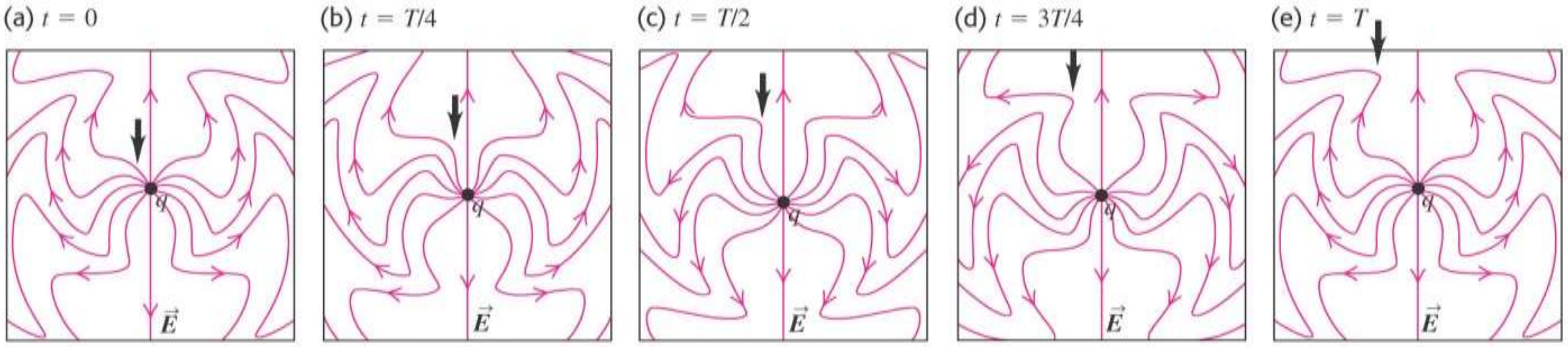
Fields can leave the sources and travel alone through space.

The bundle of electric and magnetic fields maintains itself:

If $\mathbf{B}$ were to disappear, this would produce $\mathbf{E}$; if $\mathbf{E}$ tries to go away, this would create $\mathbf{B}$.

So they propagate onward in space.

Generating Electromagnetic Radiation



Heinrich Hertz was the first person to produce electromagnetic waves intentionally in the lab

Oscillating charges in the LC circuit were sources of electromagnetic waves

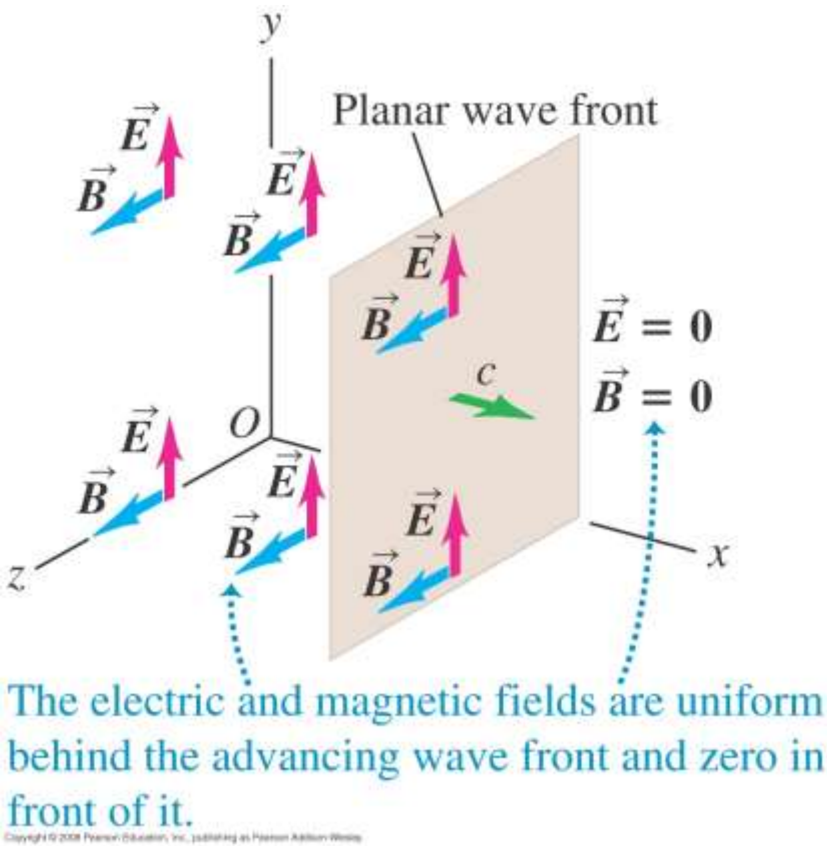
Marconi – first radio communication.
Radio transmitter- electric charges oscillate along the antennae and produce EM waves. Radio receiver – incoming EM waves induce charge oscillations and those are detected

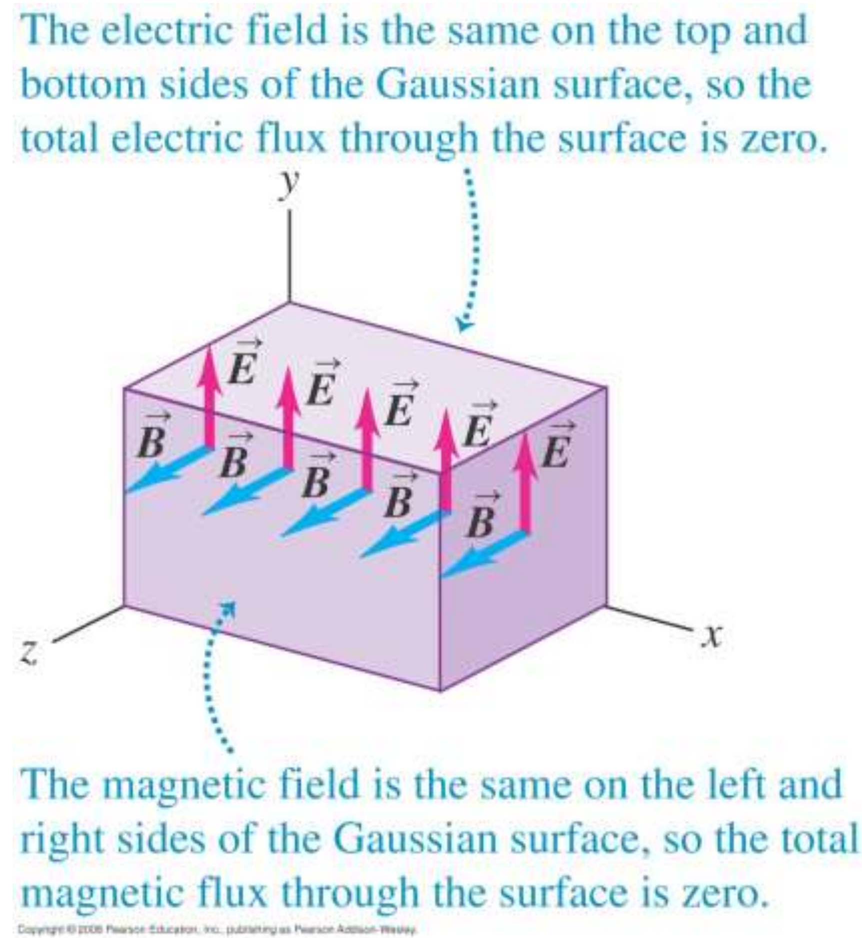
Plane EM waves

We will first show that such a plane EM wave satisfies Maxwell equations

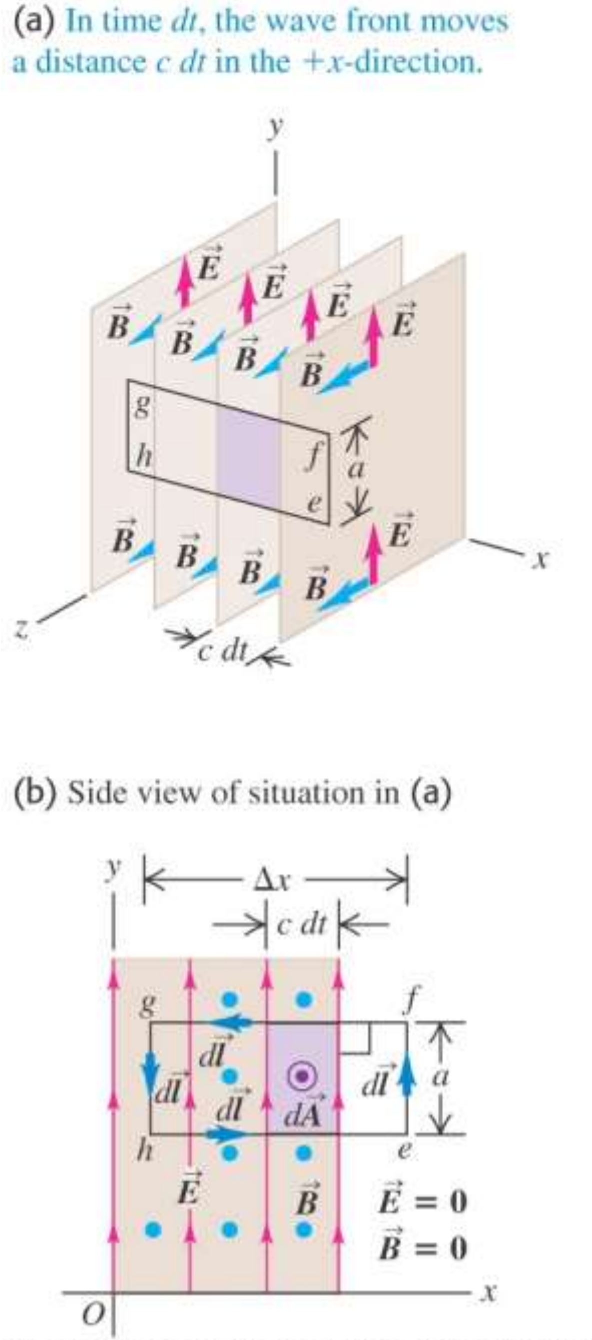
A simple plane EM wave

First, we will see if it satisfies Gauss's laws for ***E*** and ***B*** fields





Wavefront – boundary plane between the regions with and without EM disturbance



Consider Faraday's Law

$$\oint_{\Gamma} \mathbf{E} \cdot dl = -\frac{d\Phi_B}{dt}$$

Circulation of vector E around loop *efgh* equals to -*Ea*

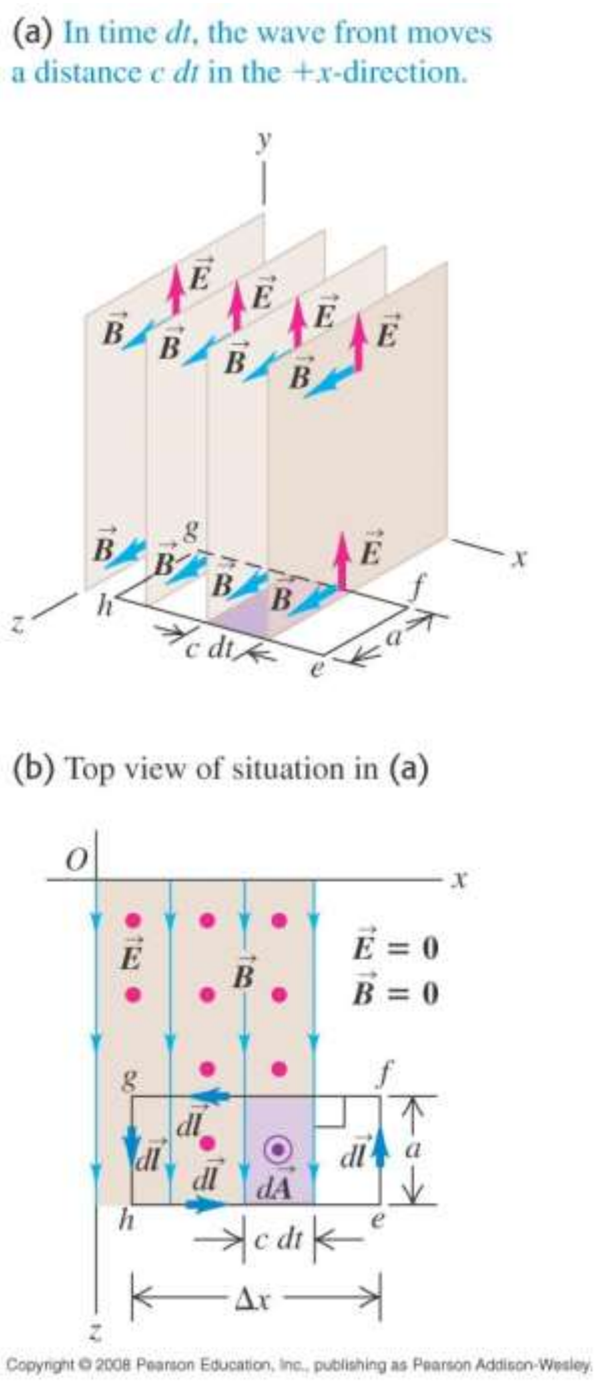
$$\oint_{\Gamma} \mathbf{E} \cdot dl = -Ea$$

Rate of change of flux through the surface bounded by *efgh* is $d\Phi=(Ba)(cdt)$

$$\frac{d\Phi_B}{dt} = Bac$$

Hence $-Ea=-Bac$, and

$$E = cB$$



Now consider Ampere's Law

$$\oint_{\Gamma} \mathbf{B} \cdot dl = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Circulation of vector ***B*** around loop *efgh* equals to *Ba*

$$\oint_{\Gamma} B \cdot dl = Ba$$

Rate of change of flux through the surface bounded by *efgh* is *dΦ=(Ea)(cdt)*

$$\frac{d\Phi_E}{dt} = Eac \qquad B = \varepsilon_0 \mu_0 c E$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad c = 299{,}792{,}458 \ m/s$$

Key Properties of EM Waves

The EM wave in vacuum is transverse; both ***E*** and ***B*** are perpendicular to the direction of propagation of the wave, and to each other. Direction of propagation and fields are related by $\vec{k} = \vec{E} \times \vec{B}$

There is definite ratio between ***E*** and ***B***; ***E*=*c*B**

The wave travels in vacuum with definite and unchanging speed *c*

Unlike mechanical waves, which need oscillating particles of a medium to transmit a disturbance, EM waves require no medium.

EM Wave Equation

Plane waves → We would be now interested in dynamic (time-varying) fields that change in space only in x-direction:

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$c^2 \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{E} = \mathbf{E}(x,t), \quad \mathbf{B} = \mathbf{B}(x,t)$$

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial E_x}{\partial x} = 0$$

$$\nabla \cdot \mathbf{B} = \frac{\partial B_x}{\partial x} = 0$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0 = -\frac{\partial B_x}{\partial t}$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = 0 = \frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$\Rightarrow E_x = B_x = 0$ for dynamic (t-dependent) fields:

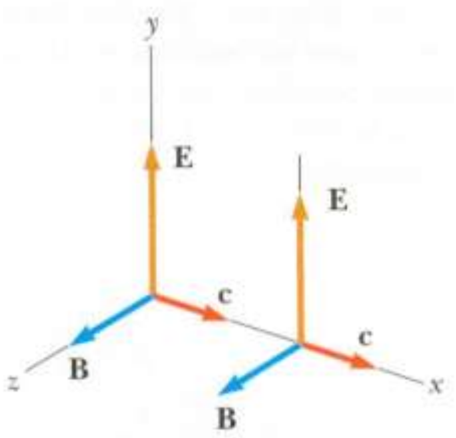
The fields are perpendicular to the direction of propagation - <u>transverse</u> waves

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$c^2 \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$



$$\mathbf{E} = \mathbf{E}(x,t), \quad \mathbf{B} = \mathbf{B}(x,t), \quad E_x = B_x = 0$$

Let us choose $E_y \neq 0, \quad E_z = 0$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0 = -\frac{\partial B_y}{\partial t}$$

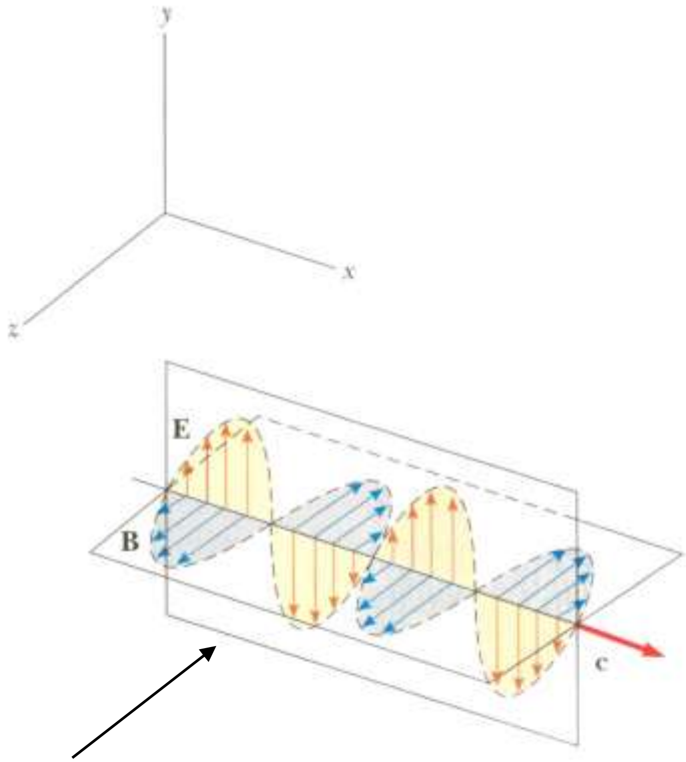
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \neq 0$$

$\Rightarrow$ Dynamic (t-dependent) $B_z \neq 0, \quad B_y = 0$

E and **B** are perpendicular to each other!

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = -\frac{\partial B_z}{\partial x} = \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

Plane waves



sinusoidal waves

Combining two equations

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial B_z}{\partial x} = \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial}{\partial x}\frac{\partial E_y}{\partial x} = -\frac{\partial}{\partial x}\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial t}\frac{\partial B_z}{\partial x} =$$

$$= \frac{1}{c^2}\frac{\partial}{\partial t}\frac{\partial E_y}{\partial t}$$

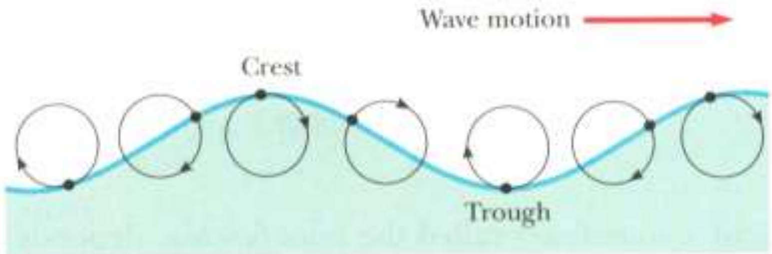
we derive the wave equation for E_y :

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

and the same for B_z :

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2} = 0$$

General (one-dimensional) wave equation



$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

describes a general wave propagation along x-direction

Here y is the wave function (e.g. displacement in the string wave motion)

v is the speed of the wave propagation

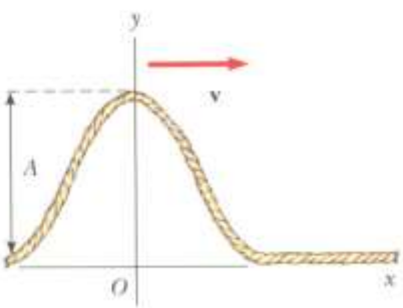
General solutions are propagating waves:

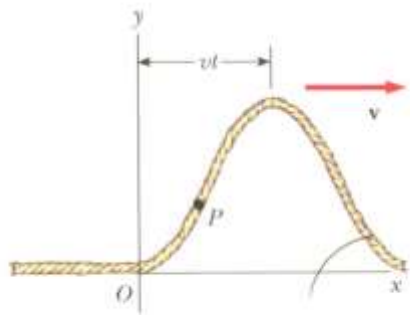
$$y = f(x - vt) + g(x + ct)$$

(f and g are arbitrary functions)

f propagates in the positive x-direction

g propagates in the negative x-direction





$$\frac{\partial y}{\partial x} = f'(x - vt), \quad \frac{\partial^2 y}{\partial x^2} = f''(x - vt)$$

$$\frac{\partial y}{\partial t} = -vf'(x - vt), \quad \frac{\partial^2 y}{\partial t^2} = v^2 f''(x - vt)$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

So our equations

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2} = 0$$

describe the propagation of EM waves

with speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \cong 3 \times 10^8$ m/s

This is the speed of light in vacuum

Light is an electromagnetic wave